CIVIL ENGINEERING

Strength of Materials

Comprehensive Theory
with Solved Examples *and* Practice Questions

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Strength of Materials

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1

Properties of Materials

1.1 INTRODUCTION

Strength of material is a branch of applied mechanics that deals with the behaviour of solid bodies subjected to various types of loading and internal forces developed due to these loading. A thorough understanding of mechanical behaviour is essential for the safe design of all structures, whether buildings, bridges, machines, motors, submarines or airplanes. Hence, strength of material is a basic subject in many engineering fields.

The objective of our analysis will be to determine the stresses, strains and deflections produced by the loads in different structures. Theoretical analysis and experimental results have equally important role in the study of strength of materials. So these quantities are found for all values of load upto the failure load, and then we will have a complete picture of the mechanical behaviour of the body.

The behaviour of a member subjected to forces depends not only on the fundamental law of Newtonian mechanics that govern the equilibrium of the forces but also on the mechanical characteristics of materials of which the member is fabricated. Sometimes, to predict the behaviour of material some necessary information regarding the characteristics of material comes from laboratory tests.

1.2 STRESS

The fundamental concept of stress can be understood by considering a prismatic bar that is loaded by axial force P at the ends as shown.

A prismatic bar is a straight structural member having constant cross-sectional area throughout its length. In the figure (a), axial force is acting away from the cross-section producing a uniform stretching of the bar, hence the bar is said to be in tension. Similarly in figure (c), axial force is acting towards the cross-section producing uniform compression of the bar, hence the bar is said to be in compression. To investigate the internal stresses produced in the bar by axial forces, we make an imaginary cut at section mn as shown in figure (b) and (d). This section is taken perpendicular to the longitudinal axis of bar. Hence it is known as cross-section.

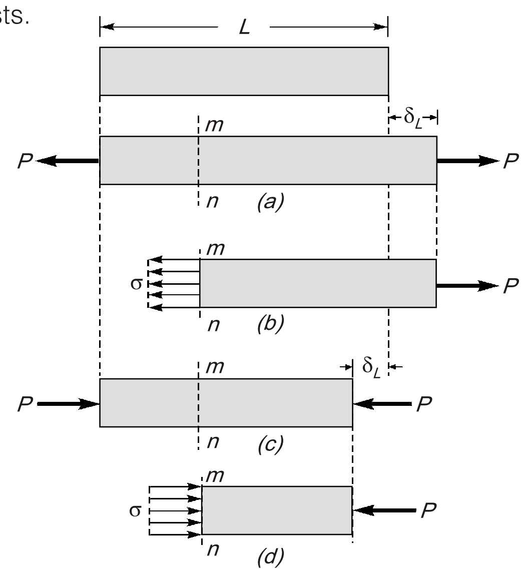


Fig. Axial stress

Now isolating the part of the bar to the right of the cut and considering the right of the cut as a free body. The force P has a tendency to move free body in the direction of load, so to restrict the motion of bar an internal force is induced which is uniformly distributed over cross-sectional area. The intensity of force developed, that is, internal force per unit area is called the **stress**.

Stress differs from pressure because pressure is defined as the externally applied force on unit area while stress is internal resistive force on unit area. To have better understanding of difference between externally applied force and internal resistance. Consider a bar suspended from a fixed end and a weight *W* is gradually applied at its free end as shown in figure.

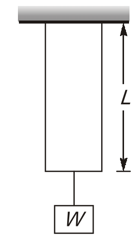
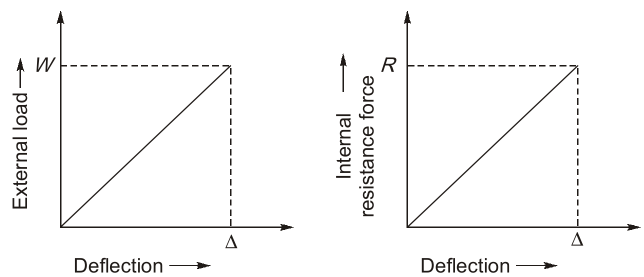


Fig. *Axial load on bar*

Case-I: Weight, W is applied gradually

Gradual loading means that value of load is zero at the starting time and gradually increases to value of *W*. Here, the bar gradually elongates with the increasing value of load. With increase in elongation, resistance forces say R will also increase gradually.

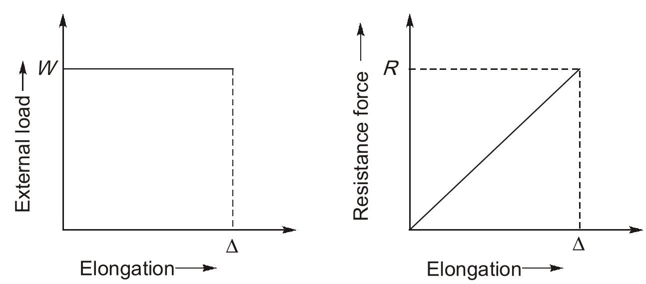


(a) Load deflection curve (b) Resistance deflection curve

Fig. *External load is applied gradually*

Case-II: Weight, W is applied suddenly

Here, external load variation with elongation of bar is such as that its value instantly increases to *W*. This sudden load will result into elongation of bar say Δ. When external load is applied suddenly, resistance force will be set up in bar, but unlike external load which is sudden, resistance force has always linear variation with elongation of bar.



(a) Load deflection curve (b) Resistance deflection curve

Fig. *External load is applied suddenly*

Now, as clear from figure (a) and (b), intensity of pressure is not equal to stress induced in bar.

Thus, stress can be defined as – **"Stress is the internal resistance of a material offered against deformation which is expressed in terms of force per unit area"**.

Stress induced in material depends upon the nature of force, point of application and cross-sectional area of material. Stress can be **tensile** or **compressive** in nature depending on the nature of load. Generally, stress is represented by the Greek letter σ. We can calculate stress mathematically as

$$\sigma = \frac{P}{A}$$

General Sign Convention:

Tensile stresses = +ve

Compressive stresses = –ve

Unit: (i) N/m^2 or Pa (SI unit)

(ii) N/mm^2 or MPa

NOTE

- Stresses are induced only when motion of bar is restricted either by some force or reaction induced. If body or bar is free to move or free expansion is allowed, then no stresses will be induced.
- Pressure has same unit but pressure is different physical quantity than stress. Pressure is external normal force distributed over surface.

On the basis of cross-sectional area considered during calculation of stresses, direct stresses can be of following two types:

(a) Engineering or nominal stress : It is the stress where the original cross-sectional area of specimen is taken.

Mathematically, $$\sigma = \frac{P}{A_0}$$

where, A_0 = Original cross-sectional area of specimen taken

(b) True or actual stress : It is the stress where the actual cross-sectional area of specimen at any time of loading is considered.

Mathematically, $$\sigma = \frac{P}{A_a}$$

where, A_a = Actual cross-sectional area of specimen at any time of loading i.e. changed area of cross-section due to loading

$A_a = A_0 \pm \Delta A$ as per our convention '+' for compression and '–' for tension is taken.

REMEMBER

- In tension, true or actual stress is always greater than engineering or nominal stress.
- In compression, true or actual stress is always less than engineering or nominal stress.

1.3 STRAIN

An axially loaded bar undergoes a change in length, becoming longer when in tension and shorter when in compression. The elongation or shortening in axially loaded member per unit length is known as strain. Strain is represented by $\in$.

Mathematically, strain can be calculated as

$$\in = \frac{\Delta L}{L}$$

Strain is dimensionless quantity and is always expressed in the form of number. If the member is in tension then the strain is called tensile strain. If the member is in compression, then the strain is called compressive strain.

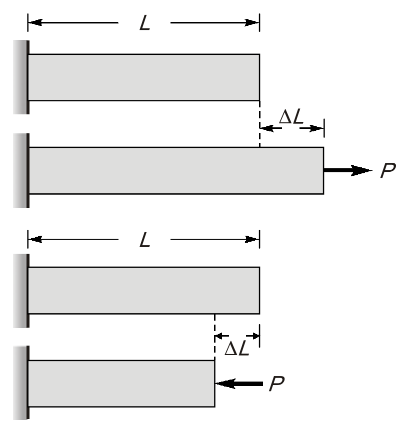


Fig. Strain in bars

On the basis of length of member used in calculation of strain, strain can be of following two types:

(a) Engineering or Nominal Strain: Engineering or nominal strain is strain calculated, when length of member is taken as original length

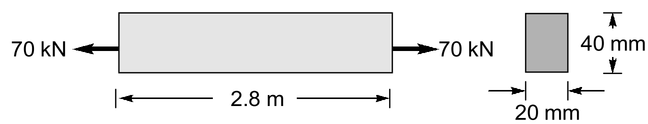
Mathematically, $$\in_0 = \frac{\Delta l}{l_0}$$ where, l_0 = original length of member

(b) True or Actual Strain : True or actual strain is strain calculated, when length of member is taken as actual length of member at loading

Mathematically, $$\in_a = \frac{\Delta l}{l_a}$$ where, l_a = Actual length of member

$l_a = l_0 \pm \Delta l$ '+' sign for tension; '–' sign for compression

Example 1.1 A prismatic bar with rectangular cross-section (20 mm × 40 mm), length L = 2.8 m is subjected to an axial tensile force of 70 kN. The measured elongation of the bar is 1.2 mm. Calculate the tensile stress and strain in the bar.



Solution:

Assuming that force acts at *CG* of section. We know that,

$$\text{Stress, } \sigma = \frac{P}{A} = \frac{70 \times 10^3 \text{N}}{20 \times 40 \text{ mm}^2} = 87.5 \text{ N/mm}^2 = 87.5 \text{ MPa}$$

and

$$\text{Strain, } \epsilon = \frac{\Delta L}{L} = \frac{1.2 \text{ mm}}{2.8 \times 1000 \text{ mm}} = 4.286 \times 10^{-4}$$

1.4 TENSILE TEST FOR MILD STEEL

The mechanical properties of materials used in engineering are determined by experiments performed on small specimen. These experiments are conducted in laboratories equipped with testing machines that are capable of loading in tension or compression. The American Society for Testing and Materials (ASTM) has published guidelines for conducting test. Tensile test is generally conducted on Universal Testing Machine (UTM).

1.4.1 General Specifications of Specimen

- Specimen is solid cylindrical rod
- Diameter of middle section 0.5” (inches)
- Gauge length 2” (inches)
- L/D ratio = 4.0

1.4.2 Stress Strain Curve for Tension

- *A* **is limit of proportionality:** Beyond this linear variation ceases. Hooke’s law is valid in OA.
- *B* **is elastic limit:** The maximum stress upto which a specimen regains its original length on removal of applied load. For mild steel, *B* is very near to *A*. However, for other materials *B* may be greater than *A*.

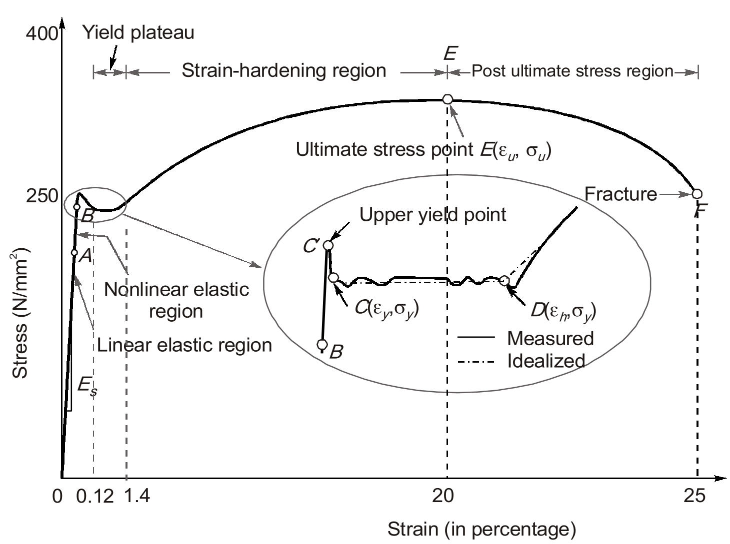


Fig. Ideal Tensile stress-strain diagram for Mild Steel

- ***C′* is upper yield point:** The magnitude of the stress corresponding to *C′* depends on the cross-sectional area, shape of the specimen and the type of the equipment used to perform the test. It has no practical significance.
- ***C* is lower yield point:** This is also called actual yield point. The stress at *C* is the yield stress (σ_y) with a typical value of σ_y = 250 N/mm^2 (for mild steel). The yielding begins at this stress.
- ***CD* represents perfectly plastic region:** It is the strain which occurs after the yielding point *C*, without any increase in stress. The strain corresponding to point *D* is about 1.4% and corresponding to *C* is about 0.12% for mild steel. Hence, plastic strain is 10 to 15 times of elastic strain.
- ***DE* represents strain hardening region:** In this range further addition of stress gives additional strain. However, strain increases with faster rate in this region. The material in this range undergoes change in its crystalline structure, resulting in increased resistance to further deformation. This portion is not used for structural design.
- ***E* is ultimate point:** The stress corresponding to this point is ultimate stress (σ_u) and the corresponding strain is about 20% for mild steel.
- ***F* is fracture point:** Stress corresponding to this is called breaking stress and strain is called fracture strain. It is about 25% for mild steel.
- ***EF* post ultimate stress region:** In this range, necking occurs, i.e. area of cross-section is drastically decreased.

NOTE

1. Strain that occurs before the yield point is called elastic strain and that which occurs after yield point with no increase in stress is called plastic strain. For mild steel, plastic strain is 10 to 15 times of elastic strain.
2. Ideal curve for tension is shown in figure. However actual behaviour is different and indicates apparently reduced yield stress in compression after strain hardening in tension. The divergence between tension and compression results is explained by Bauschinger and is called **Bauschinger effect**.

1.4.3 Actual Curve v/s Engineering Curve in Tension and Compression for Mild Steel

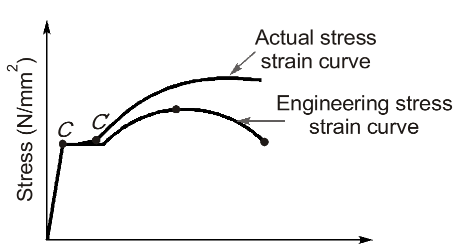


Fig. Tension curve for mild steel

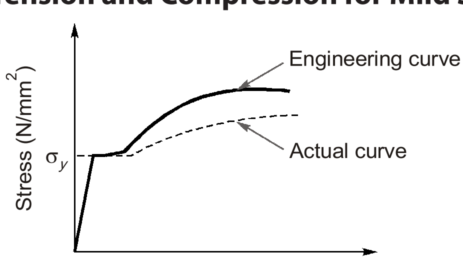


Fig. Compression curve for Mild steel

NOTE

- The fracture strain depends upon % carbon present in steel.
- With increase in percentage carbon, fracture strain reduces.
- With increase in carbon content, steel has higher yield stress and higher ultimate stresses.

- In compression, engineering stress-strain curve lies above the actual stress-strain curve, while in tension actual stress-strain curve lies above the engineering stress-strain curve.
- In compression mild steel has yield stress σ_y = 263 N/mm^2, slightly greater than tension.
- Mild steel has same Young's modulus of elasticity in compression and tension, $E = 2.1 \times 10^5$ N/mm^2.

Relation between engineering and actual stress

$$\sigma_a = \sigma_0(1 \pm \epsilon_0)$$

where, σ_a = Actual stress; σ_0 = Engineering stress; ϵ_0 = Engineering strain

As per our convention, for tension, take positive (+ve) sign and take negative (–ve) sign for compression.'

NOTE : While deriving above equation, volume changes is neglected which is true in plastic region (Non-elastic region).

1.4.4 Stress-strain Curve for other Grades of Steel in Tension

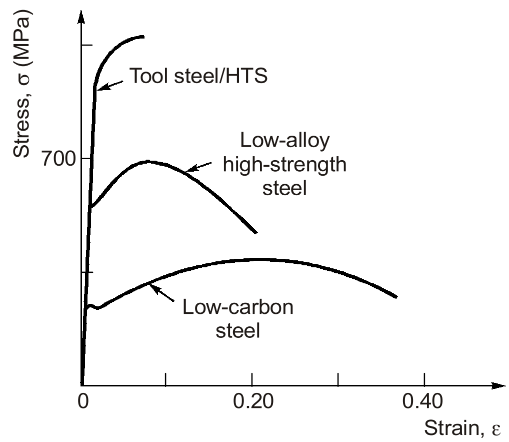


Fig. Tensile stress-strain diagram for different grades of steel

REMEMBER

- All the grades of steel have same Young's modulus of elasticity.
- Among all steel grades high tension steel (HTS) is more brittle and mild steel is more ductile.
- High tension steel has higher ultimate strength than other grades of steel.

1.4.5 Stress-strain Curve for Different Materials

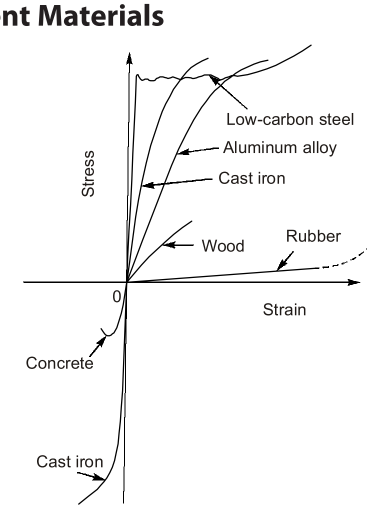


Fig. Stress-strain diagram for different material

1.5 PROPERTIES OF METALS

1.5.1 Ductility

Ductility is the property by which material can be stretched. Large deformations are thus possible in ductile materials before the absolute failure or rupture takes place. These materials have post-elastic strain (Plastic strain) greater than 5%. Some of the examples of ductile materials are mild steel, aluminium, copper, manganese, lead, nickel, brass, bronze etc.

1.5.2 Brittleness

Brittleness is the lack of ductility i.e. materials can not be stretched. In brittle materials, fracture takes place immediately after elastic limit with a relatively smaller deformation. For the brittle materials, fracture and ultimate points are same and after proportional limit very small strain is seen. Brittle materials have post elastic strain less than 5%. Some examples of brittle materials are cast iron, concrete and glass.

REMEMBER To distinguish between these two type of materials, materials with post elastic strain less than 5% at fracture point are regarded as brittle and those having post elastic strain greater than 5% at fracture point are called ductile (this value for mild steel at fracture is about 25%).

1.5.3 Malleability

Malleability is the property of metal due to which a piece of metal can be converted into a thin sheet by pressing it. A malleable material possess a high degree of plasticity. This property is of great use in operations like forging, hot rolling, drop (stamping) etc.

1.5.4 Hardness

- Hardness is resistance to scratch or abrasion.
- There are two methods of hardness measurements:
  - (a) Scratch hardness-commonly measured by Mohr's test
  - (b) Indentation hardness (abrasion) measured by
    - Brinell's hardness method
    - Rockwell hardness
    - Vickers hardness
    - Knoop hardness

1.6 CREEP

Creep is permanent deformation which is recorded with passage of time at constant loading. Total creep deformation continues to increase with time asymptotically. Consider a prismatic bar of length L on which an external static load P is applied. Due to applied static load, goes a deformation of Δ_e, but after some time it is observed that bar has gone permanent deformation and some stress developed in bar released. This effect is called creep.

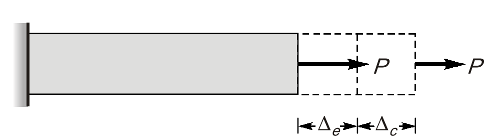


Fig. Creep in bar

where,

$$\Delta_e = \text{Elastic deflection} = \frac{PL}{AE}$$

P = Static load

Δ_c = Deformation due to creep

Factors affecting creep are as follows:

1. Magnitude of load
2. Type of loading (static or dynamic)
3. Time or age of loading
4. Temperature
    - At higher temperature, due to greater mobility of atoms most of the materials loose their strength and elastic constants also get reduced. Hence, greater deformation at elevated temperature results, even under constant loading. Therefore, creep is more pronounced at higher temperature, and thus it must be considered for design of engines and furnaces.
    - Temperature at which the creep becomes very appreciable is half of the melting point temperature on absolute scale and is known as **homologous temperature**.

1.7 STRESS RELAXATION

If a wire of metal is stretched between two immovable supports, so that it has an initial tension σ_0. The stress in the wire gradually diminishes, eventually reaching a constant value. This process, which is manifestation of creep is called **stress relaxation**. (This is the reason why electric wires sag after long time)

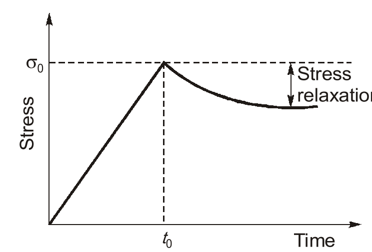


Fig. Stress relaxation

1.8 ELASTICITY

Assume for instance, we apply a tensile load to a specimen so that strain and stress follow path from *O* to *B* on stress-strain curve shown in figure. Further, when the load is removed, the material follows exactly the same curve back to the origin *O*. So, the property by which original dimensions (i.e. length and cross-section) can be recovered after unloading is known as **elasticity**.

Within elastic limit curve may be linear or non-linear. During loading, material store elastic strain energy. The total strain energy which can be stored in the given volume of the metal and can be released after unloading is called **resilience**. It is also equal to area under load deflection curve within elastic limit (*B*). When elastic limit coincides with yield point, the maximum elastic energy per unit volume is known as ***modulus of resilience***.

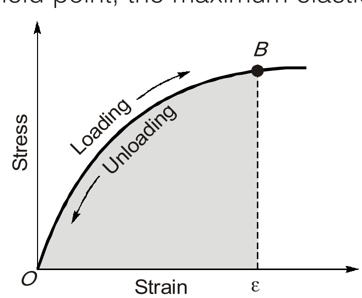


Fig. Stress strain curve

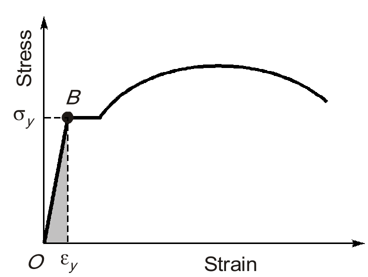


Fig. Stress strain curve in mild steel

Modulus of resilience is equal to area under the stress-strain curve within elastic limit for mild steel.

$$U_r = \frac{1}{2} \times \sigma_y \times \epsilon_y$$

NOTE

- The modulus of resilience depends upon yield strength and hence a material with higher yield strength will have higher modulus of resilience.
- Higher resilience is desirable in suspension spring and where the load absorption is required.
- HTS (High Tension Steel) has more yield stress than mild steel so it has more modulus of resilience. Thus springs are made from high tension steel.

1.8.1 Proof Stress

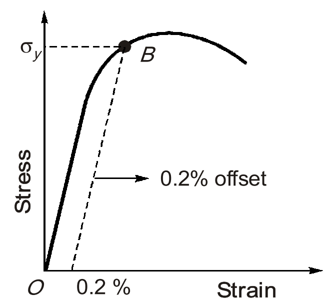


Fig. *Proof stress*

Some of the ductile metals like Aluminium (Al), Copper (Cu) and Silver (Ag) do not show clear yield point in tension test, therefore, their yield stress (σ_y) is not clearly known. For such metals, design stress is calculated by offset method. An offset of permanent plastic strain equals to 0.2% generally is marked on x-axis and a straight line is drawn which is parallel to initial portion of stress-strain curve. The point of intersection of stress-strain curve with straight line is called proof point and the corresponding stress at that point is ***called proof stress***.

1.8.2 Elasto-Plastic Behaviour of Metals

Now, suppose that material is loaded to a much higher level than elastic limit (*B*), such that point *P* is reached on stress-strain diagram. When unloading occurs the material follows path *PC* as shown in the figure, which is parallel to the initial portion of original stress-strain curve. When point *C* is reached, the load has been entirely removed but a permanent strain or residual strain *OC* remains in material. The corresponding residual elongation of the specimen is called **permanent set**.

During unloading, only *CPD* part of strain energy is recovered and is called as elastic strain energy, whereas a large part *OPC* is lost in permanent deformation and is called **inelastic strain energy**.

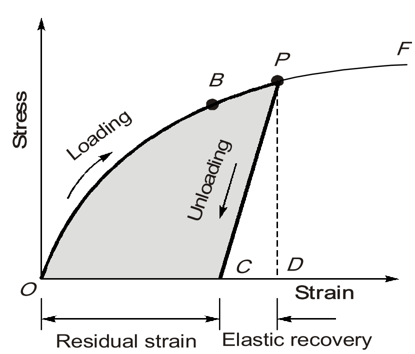


Fig. *Partially elastic behaviour*

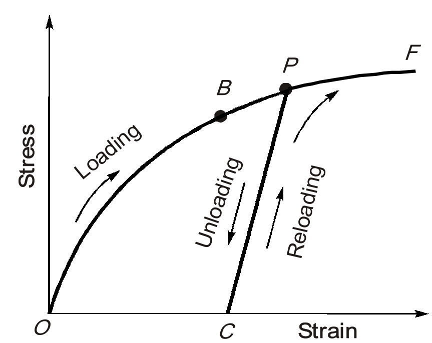
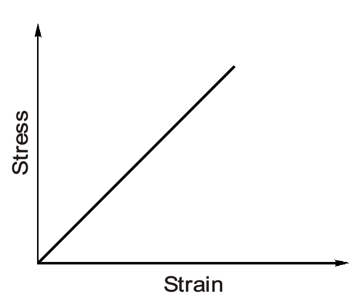


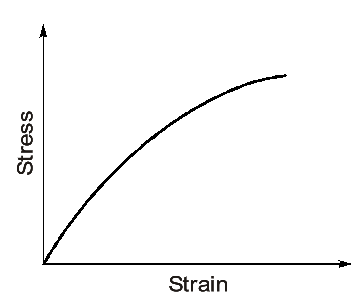
Fig. *Reloading of a material and raising of the yield stress*

If material undergoes continuous cyclic loading and unloading beyond elastic limit, then yield limit of material continuously increases. This concept is used in cold working of mild steel bar to avoid yield plateau.

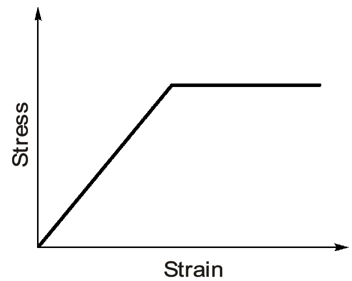
1.8.3 Types of Material Behaviour



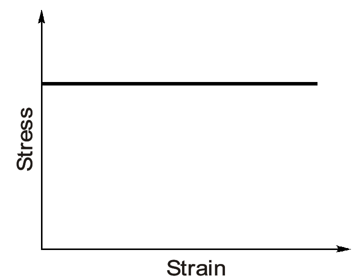
(*i*) Linear elastic



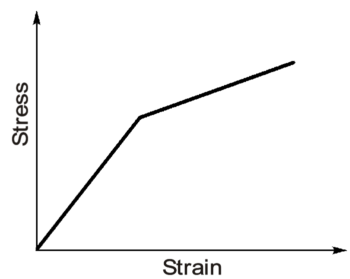
(*ii*) Non-linear elastic



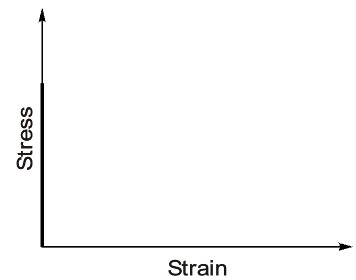
(*iii*) Elasto plastic or visco-plastic



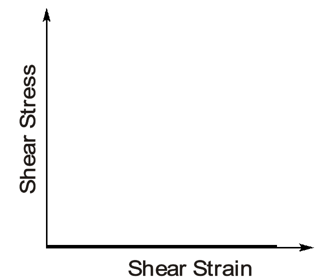
(*iv*) Perfectly plastic



(*v*) Elasto-plastic with strain hardening



(*vi*) Ideal rigid



(*vii*) Ideal fluid

1.9 TOUGHNESS

The property which enables material to absorb energy without fracture. This property is very desirable in case of cyclic loading or shock loading. If a material is tough, then it has the ability to store large strain energy before fracture. **Modulus of toughness** is total strain energy per unit volume upto fracture stage. It is equal to total area under stress-strain curve upto fracture.

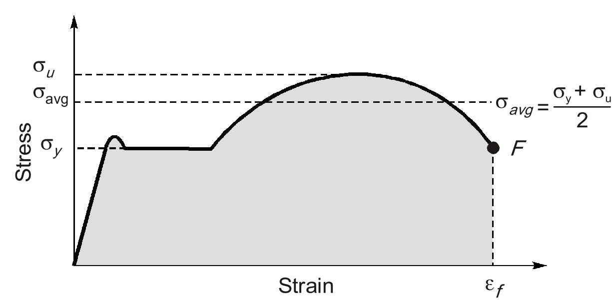


Fig. Area under stress strain upto Fracture

$$\text{Modulus of toughness} = \left(\frac{\sigma_y + \sigma_u}{2}\right) \times \epsilon_f$$

where, σ_y = yield tensile strength, σ_u = ultimate tensile strength and ϵ_f = strain at fracture point fracture.

The modulus of toughness depends upon ultimate tensile strength and strain at failure (Fracture strain). Hence, the material which is very ductile will exhibit a higher modulus of toughness as is the case with mild steel.

Remember: Ductile materials are tough and brittle materials are hard.

1.10 FATIGUE

It has been found that material behave differently under the static and dynamic loading. In cyclic or reverse cyclic loading, if total accumulated strain energy exceed the toughness, then fracture failure may occur.

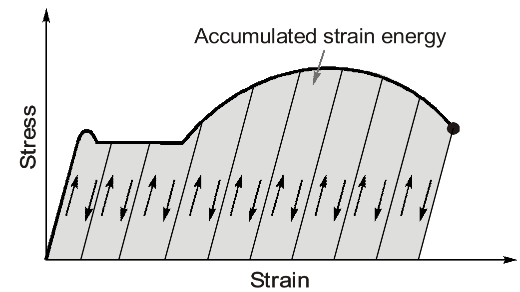


Fig. Cyclic loading

Factors affecting fatigue are:

1. Loading condition
2. Frequency of loading
3. Corrosion
4. Temperature
5. Stress concentration

The number of load cycles required to initiate surface crack is called **fatigue initiation life** and additional number of load cycle required to propagate surface crack is called **fatigue propagation life**.

To prevent fatigue failure the developed stress should be kept below endurance limit. **Endurance limit** is that stress below which a material has high probability of no failure even at infinite number of load cycles.

For, mild steel, endurance limit is 186 N/mm^2 and for Aluminium, endurance limit is 131 N/mm^2

Remember: Endurance limit is lower than the proportional limit.

Some examples of fatigue failure are:

1. Crashing of aircraft due to crack in turbine blade
2. Failure of fly wheels
3. Breaking of wire due to cyclic bending

1.11 FAILURE OF MATERIALS IN TENSION AND COMPRESSION

1.11.1 Ductile Materials in Tension Test

Ductile materials are weak in shear and failure is due to shear strain along the plane forming 45° angles with the axis of the specimen. In ductile material, cup and cone fracture take place. Failure surface is rough.

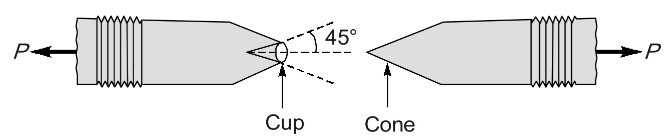


Fig. Fracture in ductile materials

NOTE : In ductile materials, necking is form before fracture.

1.11.2 Brittle Materials in Tension Test

Brittle materials are very weak in tension. Brittle materials fail due to separation of particles along the surface which is at 90° to the direction of load. Failure surface is rough.

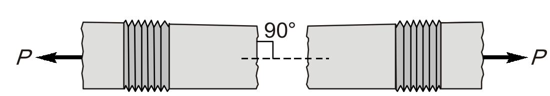


Fig. Brittle failure of material

1.11.3 Ductile Materials in Compression Test

Short compression members fail in compression yielding. Failure plane is parallel to the compressive load. In compression yielding, bulging of material occurs as shown in figure which leads to crack formation in a direction to compressive load.

OBJECTIVE BRAIN TEASERS

Q.1 Match **List-I (Type of material)** with **List-II (Characteristics)** and select the correct answer using the codes given below the lists:

List-I
A. Elastic material
B. Rigid material
C. Plastic material
D. Resilient material

List-II
1. Does not store energy
2. Has no plastic region in stress strain curve
3. Behave as a spring
4. Offers resistance to deformation
5. Does not offer resistance to deformation

Codes:

	A	B	C	D
(a)	4	1	5	3
(b)	4	1	5	2
(c)	2	3	4	5
(d)	1	3	4	5

Q.2 A structure is said to be linearly elastic if
(a) Load ∝ displacement
(b) $\text{Load} \propto \dfrac{1}{\text{Displacement}}$
(c) Energy ∝ displacement
(d) Energy ∝ Load

Q.3 Which one is not the characteristics of fatigue fracture?
(a) Rough fracture surface
(b) Rough and smooth areas on fracture surface
(c) Plastic deformation
(d) Conchoidal markings on fracture surface

Q.4 Stress curve is always a straight line for
(a) elastic material
(b) materials obeying Hooke's law
(c) elasto plastic materials
(d) none of the above

Q.5 Consider the following statements:
The principle of superposition is applied to
1. Linear elastic bodies
2. Bodies subjected to small deformations

Which of these statements is/are correct?
(a) 1 alone
(b) 1 and 2
(c) 2 alone
(d) neither 1 nor 2

Q.6 The strain at a point is a
(a) Scalar
(b) Vector
(c) Tensor
(d) None of these

Q.7 Consider the following statements
1. Strength of steel increases with carbon content
2. Young's modulus of steel increase with carbon content
3. Young's modulus of steel remain unchanged with variation of carbon content

Which of these statements is/are correct?
(a) 1 only
(b) 2 only
(c) 1 and 2
(d) 1 and 3

Q.8 True stress σ is related with conventional stress σ_0 as
(a) $\dfrac{\sigma}{\sigma_0} = (1+\epsilon)^2$
(b) $\dfrac{\sigma}{\sigma_0} = \dfrac{1}{(1+\epsilon)^2}$
(c) $\dfrac{\sigma}{\sigma_0} = \dfrac{1}{(1+\epsilon)}$
(d) $\dfrac{\sigma}{\sigma_0} = 1+\epsilon$

Q.9 Steel has its yield strength of 400 N/mm^2 and modulus of elasticity of 2×10^5 MPa. Assuming the material to obey Hooke's law up to yielding, what is its proof resilience?
(a) 0.8 N/mm^2
(b) 0.4 N/mm^2
(c) 0.6 N/mm^2
(d) 0.7 N/mm^2

Q.10 What would be the shape of the failure surface of a standard cast iron specimen subjected to torque?
(a) Cup and cone shape at the center.
(b) Plane surface perpendicular to the axis of the specimen.
(c) Pyramid type wedge-shaped surface perpendicular to the axis of the specimen.
(d) Helicoidal surface at 45° to the axis of the specimen.

ANSWER KEY

1. (a)	2. (a)	3. (a)	4. (b)	5. (b)
6. (c)	7. (d)	8. (d)	9. (b)	10. (d)
11. (d)	12. (c)	13. (b)	14. (d)	15. (b)
16. (a)	17. (a)	18. (d)	19. (c)	

HINTS & EXPLANATIONS

4. (b)

Stress ∝ strain (Hooke's law)

Which is valid within proportional limit.

Within elastic limit stress – strain curve may be linear or nonlinear. For e.g. Rubber.

7. (d)

Strength of steel increases with carbon content but Young's modulus remains constant.

9. (b)

$$\text{Proof resilience} = \frac{\sigma_y^2}{2E} = \frac{400^2}{2\times2\times10^5} = 0.4\text{N/mm}^2$$

10. (d)

Brittle materials fails in a plane at 45° from the axis when subjected to torque because they are weak in tension compare to shear. If ductile materials are subjected to torque, then the failure surface will be in a plane at 90° from the axis of shaft.

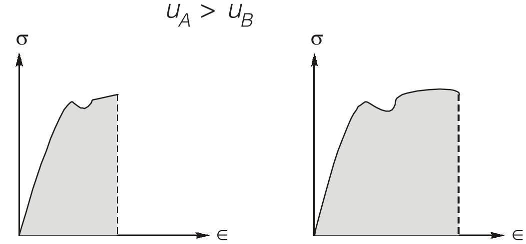
12. (c)

At higher temperature creep become more important because after temperature half of melting point, it becomes uncontrollable.

The nature of creep is elastic as well as plastic.

13. (b)

$$u_A > u_B$$



For material A For material B

Strain energy per unit volume of *A* is less than *B*.

$$\frac{u_A}{V_A} < \frac{u_B}{V_B}$$

Since materials are same size, $\frac{u_A}{A \cdot L_A} = \frac{u_B}{A \cdot L_B}$

$$\therefore \quad L_A > L_B$$

and given that $\sigma_{y_A} > \sigma_{y_B}$

∴ Material *B* is more ductile than that of *B*.

19. (c)

The two elastic constants for isotropic materials are usually expressed as Young's modulus and the Poisson's ratio. However the other elastic constants *K*, *G* can also be used. For isotropic material, *G* and *K* are found out from *E* and μ. All metals at micro level are anisotropic.

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CONVENTIONAL BRAIN TEASERS

Q.1 For a brass alloy, the following engineering stresses produce the corresponding plastic engineering strains, prior to necking :

Engineering Stress (MPa)	Engineering Strain
235	0.194
250	0.296

On the basis of this information, compute the engineering stress necessary to produce an engineering strain of 0.25.

Solution :

(i) As per given information, we first need to convert engineering stresses and strains to true stresses and strains.

True stress, $\sigma_T = \sigma(1 + \varepsilon)$

$$(\sigma_T)_1 = \sigma_1(1 + \varepsilon) = 235(1 + 0.194) = 280.59 \text{ MPa}$$

True strain, $\varepsilon_T = \ln(1 + \varepsilon)$

$$(\varepsilon_T)_1 = \ln(1 + \varepsilon_1) = \ln(1 + 0.194) = 0.1773$$

Similarly, true stress $(\sigma_{T2}) = \sigma_2(1 + \varepsilon_2) = 250(1 + 0.296) = 324 \text{ MPa}$

True strain, $(\varepsilon_T)_2 = \ln(1 + \varepsilon_2) = \ln(1 + 0.296) = 0.259$

As we know, true stress and strain relationship given by :

$$\sigma_T = K\varepsilon_T^n$$

$$\ln \sigma_T = \ln(K) + n \ln(\varepsilon_T)$$

$$\ln(280.59) = \ln(K) + n \ln(0.1773) \quad \text{...(i)}$$

$$\ln(324) = \ln(K) + n \ln(0.259) \quad \text{...(ii)}$$

By solving equation (i) and (ii) respectively :

$$n = 0.379$$

$$\ln(K) = 6.2935$$

$$K = 541.0436 \text{ MPa}$$

General equation, $\sigma_T = K\varepsilon_T^n$

$$\sigma_T = 541.0436\,(\varepsilon_T)^{0.379}$$

For engineering strain, $\varepsilon = 0.25$

True strain, $\varepsilon_T = \ln(1 + 0.25) = 0.223$

$$\sigma_T = 541.0486(0.223)^{0.379}$$

$$\sigma_T = 306.3654 \text{ MPa}$$

$$\sigma_T = \sigma(1 + \varepsilon)$$

$$306.3654 = \sigma(1 + 0.25)$$

Required engineering stress,

$$\sigma = \frac{306.3654}{1.25} = 245.092 \text{ MPa}$$

Q.2 Prove that for maximum strain hardening ,true strain is equal to work hardening coefficient. For a material true stress-strain curve follows the relationship $s_T = s_0 + k\hat{I}_T^n$. If true stress, $s_T = 360$ MPa, flow stress at zero plastic strain, $s_0 = 250$ MPa, strain hardening exponent, $n = 0.28$ and true plastic strain $\hat{I}_T = 0.07$. Determine the value of $\frac{d\sigma_T}{d\in_T}$.

Solution :

$$\text{True strain} = \int_0^{\in} d\in = \int_{l_0}^{l} \frac{dl}{l}$$

True strain, $\in = [\ln l]_{l_0}^{l} = \ln\left(\frac{l}{l_0}\right)$

$$\left(\frac{l}{l_0} - 1\right) + 1 = e + 1$$

$$\in = \ln(1 + e) \qquad \text{(where, } e \text{ is engineering strain)}$$

True stress: $\sigma_f = \frac{P}{A} \times \frac{A_0}{A_0}$

$$\sigma_f = \sigma_0 \times \frac{A_0}{A}$$